

The GED Math Foundations Roadmap

Installment 5 — Order, Signs, Powers & Roots

Lessons 14–17: Order of Operations · Positive & Negative Numbers · The Rules of Exponents · Square Roots

Start anywhere

Each lesson stands completely on its own — you don't need the one before it. Pick the topic you want to work on today and dive in. Every lesson tells you the idea first, shows you a worked example, gives you practice, and then explains every answer.

Every lesson speaks in the same five colors — after one lesson, you can find your place by color alone:

blue = the idea	green = worked examples	amber = your turn	violet = answers, explained	teal = more practice
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Lesson 14: Order of Operations (PEMDAS)

The Traffic Laws of Math

LESSON 14 OF 19	PHASE 2 · THE CORE FOUR	ABOUT 20-25 MINUTES
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Warm-Up — start with a win

Two questions you can already do. Answer them in your head — then you've officially started.

1. What is 2×5 ?
2. What is $(1 + 2) \times 2$?

Answers: 10 · 6 — the parentheses went first, and you knew it

The Concept — the idea behind the math

When an expression mixes several operations, you can't just work left to right and hope. Without an agreed order, $3 + 4 \times 2$ could be 14 or

11 depending on who's doing it. Math settles the argument with a fixed order of operations, often remembered as **PEMDAS**:

- **P** — Parentheses (and anything grouped) first
- **E** — Exponents next
- **M and D** — Multiplication and Division, **together, left to right**
- **A and S** — Addition and Subtraction, **together, left to right**

The trap that catches everyone: M and D are the **same** step — you don't do all the multiplying and then all the dividing; you take them as they come, left to right. Same with adding and subtracting. “My Dear Aunt Sally” makes it sound like four steps, but it's really two.

Think of it as traffic laws. The rules aren't there to be fussy — they're there so everyone arrives at the same answer.

Worked Examples — watch the moves

$3 + 4 \times 2$ — multiplication outranks addition:

$$4 \times 2 = 8, \text{ then } 3 + 8 = 11 \quad (\text{not } 14)$$

$(3 + 4) \times 2$ — parentheses change everything:

$$3 + 4 = 7, \text{ then } 7 \times 2 = 14$$

$20 \div 4 \times 5$ — division and multiplication, left to right:

$$20 \div 4 = 5, \text{ then } 5 \times 5 = 25 \quad (\text{not } 1)$$

$2 + 3^2 \times 4$ — exponent, then multiply, then add:

$$3^2 = 9, \text{ then } 9 \times 4 = 36, \text{ then } 2 + 36 = 38$$

Check for Understanding — your turn (5 questions)

1. $5 + 2 \times 3 = ?$

2. $(5 + 2) \times 3 = ?$

3. $4^2 - 6 = ?$

4. $12 \div 2 \times 3 = ?$

5. **Critical Thinking:** $2 + 3 \times (8 - 5)^2 = ?$

PAUSE HERE — try all five on your own first. Getting them wrong costs nothing; that's what they're for.

Answer Key — every answer, explained

1. **11** (Multiply first: $2 \times 3 = 6$, then $5 + 6$.)

2. **21** (Parentheses first: $5 + 2 = 7$, then 7×3 .)

3. **10** (Exponent first: $4^2 = 16$, then $16 - 6$.)

4. **18** (Left to right: $12 \div 2 = 6$, then 6×3 . Not $12 \div 6 = 2$.)

5. **29** (Parentheses: $8 - 5 = 3$. Exponent: $3^2 = 9$. Multiply: $3 \times 9 = 27$. Add: $2 + 27$.)

More Practice — reps make it stick

Evaluate each expression:

1. $6 + 4 \times 2$

2. $(6 + 4) \times 2$

3. $3 \times 5 - 4$

4. $18 \div 3 + 1$

5. $2^3 + 1$

6. $20 - 4 \times 3$

7. $16 \div 2 \times 4$

8. $(4 + 2)^2 \div 9$

More Practice — Answer Key

1. **14** ($4 \times 2 = 8$, then $6 + 8$.)
2. **20** ($6 + 4 = 10$, then $\times 2$.)
3. **11** ($3 \times 5 = 15$, then $- 4$.)
4. **7** ($18 \div 3 = 6$, then $+ 1$.)
5. **9** ($2^3 = 8$, then $+ 1$.)
6. **8** ($4 \times 3 = 12$, then $20 - 12$.)
7. **32** (Left to right: $16 \div 2 = 8$, then $\times 4$.)
8. **4** (Parentheses: 6; square: 36; then $36 \div 9$.)

Update your tracker:

Topic	New to this	Practicing	Got this!
The PEMDAS Order	☐	☐	☐
Left-to-Right for $\times \div$ and $+-$	☐	☐	☐

Lesson 15: Positive & Negative Numbers

Which Way on the Number Line

LESSON 15 OF 19

PHASE 2 · BELOW ZERO

ABOUT 20-25 MINUTES

Warm-Up — start with a win

Two questions you can already do. Answer them in your head — then you've officially started.

1. It's 5° outside and the temperature drops 5 degrees. What's the new temperature?
2. Which is colder: -10° or -2° ?

Answers: 0° · -10°

The Concept — the idea behind the math

Picture a number line. Zero sits in the middle; positive numbers run to the right, negative numbers to the left. A negative number isn't "less than nothing" — it's a direction. Think of money: $+20$ is twenty dollars in your pocket, -20 is twenty dollars you owe.

Adding:

- **Same signs** — add the numbers and keep the sign. $(-4) + (-5) = -9$.
- **Different signs** — subtract the smaller from the larger, and take the sign of the bigger one. $-7 + 3$: subtract to get 4, and since 7 is the bigger and it's negative, the answer is -4 .

Subtracting — the key move: subtracting is the same as **adding the opposite**. So $a - b$ becomes $a + (-b)$. And subtracting a negative turns into adding: $5 - (-3) = 5 + 3 = 8$. (Two "minus" signs back to back face you forward again — like turning around twice.)

Multiplying and dividing — just watch the signs: same signs give a **positive** answer; different signs give a **negative** one. $(-6) \times (-2) = 12$; $(-6) \times 2 = -12$.

Worked Examples — watch the moves

$-7 + 3$ — different signs: $7 - 3 = 4$, sign of the bigger (7, negative):

$$-7 + 3 = -4$$

$5 - (-3)$ — subtracting a negative becomes adding:

$$5 - (-3) = 5 + 3 = 8$$

$(-6) \times (-2)$ — same signs, positive:

$$(-6) \times (-2) = 12$$

Check for Understanding — your turn (5 questions)

1. $-8 + 5 = ?$
2. $-3 + (-6) = ?$
3. $4 - 9 = ?$
4. $(-5) \times (-4) = ?$
5. **Critical Thinking:** The temperature is -6°F and then drops another 4 degrees. What is the temperature now?

PAUSE HERE — try all five on your own first. Getting them wrong costs nothing; that's what they're for.

Answer Key — every answer, explained

1. **-3** (Different signs: $8 - 5 = 3$, and 8 is the bigger, so negative.)
2. **-9** (Same signs: add to 9, keep the negative.)
3. **-5** ($4 - 9 = -(9 - 4) = -5$; you cross below zero.)
4. **20** (Same signs multiplied give a positive.)
5. **-10°F** (Dropping means subtracting: $-6 - 4 = -10$.)

More Practice — reps make it stick

Add or subtract:

1. $-2 + 7$
2. $-10 + (-4)$
3. $6 - 10$
4. $3 - (-5)$
5. $-7 - 2$

Multiply or divide:

6. -8×3
7. $(-20) \div (-5)$
8. $-15 \div 3$

More Practice — Answer Key

1. **5** (Different signs: $7 - 2 = 5$, bigger is positive.)
2. **-14** (Same signs: add, keep negative.)
3. **-4** ($6 - 10$ crosses below zero.)
4. **8** (Minus a negative becomes plus: $3 + 5$.)
5. **-9** ($-7 - 2 = -7 + (-2)$; same signs.)
6. **-24** (Different signs give a negative.)
7. **4** (Same signs give a positive.)
8. **-5** (Different signs give a negative.)

Update your tracker:

Topic	New to this	Practicing	Got this!
Adding & Subtracting Signed Numbers	☐	☐	☐

Topic	New to this	Practicing	Got this!
Multiplying & Dividing Signed Numbers	☐	☐	☐

Lesson 16: The Rules of Exponents

Nine Rules, One Idea: Repeated Multiplication

LESSON 16 OF 19

PHASE 5 · POWERS &
ROOTS

ABOUT 35-45 MINUTES

Warm-Up — start with a win

Two questions you can already do. Answer them in your head — then you've officially started.

1. $2 \times 2 \times 2 = ?$
2. Write 4×4 using an exponent.

Answers: 8 · 4^2 — the little number counts the 4s

The Concept — the idea behind the math

An exponent is just a shorthand for repeated multiplication. 2^3 means $2 \times 2 \times 2 = 8$. The **base** (2) is what gets multiplied; the **exponent** (3) counts how many times. Every rule below is a shortcut that falls right out of that one idea — you never have to memorize them blindly; you can always rebuild them by writing the multiplication out.

This is the longest lesson in the roadmap — plan on a full sitting, or split it: rules 1-5 one day, rules 6-9 the next. The tracker at the end has a row for each half.

1. Product Rule (add the exponents). Same base, multiplied: keep the base, add the exponents. $x^a \cdot x^b = x^{a+b}$

$$2^2 \cdot 2^3 = 2^5 = 32 \quad (\text{because } (2 \times 2)(2 \times 2 \times 2) \text{ is five 2's})$$

2. Quotient Rule (subtract the exponents). Same base, divided: keep the base, subtract the exponents. $x^a \div x^b = x^{a-b}$

$$5^5 \div 5^2 = 5^3 = 125$$

3. Power to a Power (multiply the exponents). A power raised to another power: multiply the exponents. $(x^a)^b = x^{a \times b}$

$$(3^2)^3 = 3^6 = 729$$

4. Power over Factors (distribute to each factor). A product inside parentheses: the power lands on each factor. $(x \cdot y)^a = x^a \cdot y^a$

$$(2 \cdot 5)^2 = 2^2 \cdot 5^2 = 4 \cdot 25 = 100$$

5. Power over a Fraction (distribute to top and bottom). A fraction inside parentheses: the power lands on numerator and denominator. $(x/y)^a = x^a / y^a$

$$(2/3)^2 = 2^2/3^2 = 4/9$$

6. Raising to the First Power. Anything to the first power is just itself. $x^1 = x$

$$7^1 = 7$$

7. Zero Exponent Rule. Any nonzero base to the zero power equals 1. $x^0 = 1$

$$9^0 = 1 \quad (\text{because } x^3 \div x^3 = x^0, \text{ but it's also clearly } 1)$$

8. Negative Exponents (flip to make it positive). A negative exponent means “take the reciprocal, then the exponent turns positive.” $x^{-a} = 1 / x^a$

Put another way: a factor crossing the fraction bar — top to bottom or bottom to top — flips the sign of its exponent.

$$2^{-3} = 1/2^3 = 1/8 \quad \text{and} \quad 1 / 3^{-2} = 3^2 = 9$$

9. Fractional Exponents (the bottom is a root, the top is a power). The denominator tells you which root to take; the numerator tells you which power to raise to. $x^{1/n}$ = the n-th root of x, and $x^{m/n} = (\text{n-th root of } x)^m = \text{n-th root of } (x^m)$.

$$9^{1/2} = \sqrt{9} = 3$$

This last rule earns extra room — see the next box.

Worked Examples — watch the moves

Fractional Exponents — Either Order Works (Pick the Easy

One)

With a fractional exponent like $x^{m/n}$, the bottom number is a root and the top is a power — and **you may carry them out in either order**. The answer is identical both ways:

$$x^{m/n} = (n\sqrt{x})^m = n\sqrt{(x^m)}$$

The real skill is choosing the order that keeps you on clean whole numbers. Which way that is depends on the numbers in front of you — here are three tactics:

- **Simplify the exponent first if it reduces.** In $2^{4/2}$, the exponent $4/2$ is just 2, so $2^{4/2} = 2^2 = 4$ — and you're done.
- **If the base is a perfect root, take the root first.** $8^{2/3}$: the cube root of 8 is a clean 2, then $2^2 = 4$. (Going the other way would make you find the cube root of 64.)
- **If taking the root would give an ugly decimal, do the power first.** The square root of 2 is messy, so for $2^{4/2}$ go $2^4 = 16$, then $\sqrt{16} = 4$ — every step a whole number.

Watch two problems pull in opposite directions:

$8^{2/3}$ — the base is a perfect cube, so **root first** is cleanest:

$$\sqrt[3]{8} = 2, \text{ then } 2^2 = 4 \quad (\text{power first works too, but makes you find } \sqrt[3]{64})$$

$2^{4/2}$ — base 2 is not a perfect square, so root-first hits the messy $\sqrt{2}$; go **power first** instead:

$$2^4 = 16, \text{ then } \sqrt{16} = 4 \quad (\text{or simplest of all: } 4/2 = 2, \text{ so } 2^2 = 4)$$

So there's **no single “always.”** Look at the numbers and take the route that stays on whole numbers — that judgment is the part worth practicing, because it's what works without a calculator. (Two easy endpoints: when the top is 1 it's a pure root; when the bottom is 1 it's a pure power.)

Check for Understanding — your turn (5 questions)

1. Use the Product Rule: $3^4 \cdot 3^2 = ?$
2. Use the Quotient Rule: $7^5 \div 7^3 = ?$
3. Use the Zero Exponent Rule: $12^0 = ?$
4. Rewrite with a positive exponent, then evaluate: $5^{-2} = ?$
5. **Critical Thinking:** Evaluate $27^{2/3}$ both ways — root first and power first — and say which was easier.

PAUSE HERE — try all five on your own first. Getting them wrong costs nothing; that's what they're for.

Answer Key — every answer, explained

1. **$3^6 = 729$** (Product Rule: add $4 + 2 = 6$.)
2. **$7^2 = 49$** (Quotient Rule: subtract $5 - 3 = 2$.)
3. **1** (Zero Exponent Rule: any nonzero base to the 0 is 1.)
4. **$1/25$** (Negative Exponent: $5^{-2} = 1/5^2 = 1/25$.)
5. **9** (Root first: $\sqrt[3]{27} = 3$, then $3^2 = 9$. Power first: $27^2 = 729$, then $\sqrt[3]{729} = 9$. Root first is far easier here, because 27 is a perfect cube — the root collapses to a single digit.)

More Practice — reps make it stick

Simplify or evaluate. (Leave answers as a single power where it's a variable.)

1. $2^3 \cdot 2^4$
2. $x^5 \cdot x^2$
3. $6^8 \div 6^5$
4. $(2^3)^2$

5. $(4 \cdot 3)^2$

6. $(2/5)^2$

7. 100^0

8. 3^{-2}

9. $9^{1/2}$

10. $8^{4/3}$

11. $2^{4/2}$

More Practice — Answer Key

1. $2^7 = 128$ (Product Rule: $3 + 4$.)

2. x^7 (Product Rule: $5 + 2$; the base stays x .)

3. $6^3 = 216$ (Quotient Rule: $8 - 5$.)

4. $2^6 = 64$ (Power to a Power: 3×2 .)

5. 144 (Power over Factors: $4^2 \cdot 3^2 = 16 \cdot 9$.)

6. $4/25$ (Power over a Fraction: $2^2/5^2$.)

7. 1 (Zero Exponent Rule.)

8. $1/9$ (Negative Exponent: $1/3^2$.)

9. 3 (Fractional Exponent: $\sqrt{9}$. Pure root — the top is 1.)

10. 16 (Fractional Exponent, root first: $\sqrt[3]{8} = 2$, then $2^4 = 16$ — here 8 is a perfect cube, so root first stays clean.)

11. 4 (Root first here hits the messy $\sqrt{2}$. So simplify the exponent — $4/2 = 2$, giving $2^2 = 4$ — or go power first: $2^4 = 16$, then $\sqrt{16} = 4$.)

Update your tracker:

Topic	New to this	Practicing	Got this!
Product, Quotient & Power Rules (1-5)	☐	☐	☐

Topic	New to this	Practicing	Got this!
Zero & Negative Exponents (6–8)	☐	☐	☐
Fractional Exponents / Roots (9)	☐	☐	☐

Lesson 17: Square Roots

Undoing a Square

LESSON 17 OF 19

PHASE 5 · POWERS &
ROOTS

ABOUT 20-25 MINUTES

Warm-Up — start with a win

Two questions you can already do. Answer them in your head — then you've officially started.

1. What number times itself gives 9?
2. $10 \times 10 = ?$

Answers: 3 · 100 — so the square root of 100 is 10, and you're halfway home

The Concept — the idea behind the math

Squaring a number means multiplying it by itself: $5^2 = 25$. A **square root** runs that backward and asks: *what number, times itself, gives me this?* $\sqrt{25} = 5$, because $5 \times 5 = 25$. The square-root symbol $\sqrt{}$ is called a radical.

A picture helps: if a square has an **area** of 25, then the length of each **side** is $\sqrt{25} = 5$. Square roots are how you get from area back to a side length.

The perfect squares are worth knowing by heart — they make these instant:

1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144 (that's 1^2 through 12^2)

For numbers in between, estimate by finding the two perfect squares it falls between. $\sqrt{50}$ sits just past $\sqrt{49} = 7$, so it's a little more than 7.

And the bridge to Lesson 16: a square root *is* a fractional exponent. $\sqrt{x} = x^{1/2}$. They're two names for the same operation.

Worked Examples — watch the moves

$$\sqrt{36} = 6 \quad (\text{because } 6 \times 6 = 36)$$

$$\sqrt{81} = 9 \quad (\text{because } 9 \times 9 = 81)$$

Estimate $\sqrt{20}$. It falls between $\sqrt{16} = 4$ and $\sqrt{25} = 5$, closer to the 4 end — about 4.5.

Check for Understanding — your turn (5 questions)

1. $\sqrt{49} = ?$
2. $\sqrt{100} = ?$
3. $\sqrt{64} = ?$
4. Between which two whole numbers does $\sqrt{30}$ fall?
5. **Critical Thinking:** A square garden has an area of 144 square feet. How long is each side?

PAUSE HERE — try all five on your own first. Getting them wrong costs nothing; that's what they're for.

Answer Key — every answer, explained

1. **7** ($7 \times 7 = 49$.)
2. **10** ($10 \times 10 = 100$.)
3. **8** ($8 \times 8 = 64$.)
4. **Between 5 and 6** ($\sqrt{25} = 5$ and $\sqrt{36} = 6$, and 30 is between 25 and 36.)
5. **12 feet** ($\sqrt{144} = 12$, since $12 \times 12 = 144$ — area back to a side length.)

More Practice — reps make it stick

Find each square root:

1. $\sqrt{9}$
2. $\sqrt{16}$
3. $\sqrt{121}$
4. $\sqrt{1}$

Estimate and reason:

5. Between which two whole numbers is $\sqrt{50}$?
6. Between which two whole numbers is $\sqrt{8}$?
7. A square rug covers 81 square feet. How long is each side?
8. Write $\sqrt{25}$ as a power (using a fractional exponent).

More Practice — Answer Key

1. **3** ($3 \times 3 = 9$.)
2. **4** ($4 \times 4 = 16$.)
3. **11** ($11 \times 11 = 121$.)
4. **1** ($1 \times 1 = 1$.)
5. **Between 7 and 8** ($\sqrt{49} = 7$, $\sqrt{64} = 8$.)
6. **Between 2 and 3** ($\sqrt{4} = 2$, $\sqrt{9} = 3$.)
7. **9 feet** ($\sqrt{81} = 9$.)
8. **$25^{1/2}$** (A square root is the $1/2$ power — the link back to Lesson 16.)

Update your tracker:

Topic	New to this	Practicing	Got this!
Perfect Squares & Square Roots	☐	☐	☐
Estimating Square Roots	☐	☐	☐

